(22)

2019

STATISTICS — GENERAL

Paper : CC/GE – 2

Unit : I – III

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** 1 and 2, and ***any two*** from the rest.

1. Answer ***any ten*** of the following : 1×10

(a) A single letter is selected at random from the word 'BINOMIAL'. Find the probability that it is a vowel.

(b) If two events are independent, check whether they are mutually exclusive or not.

(c) For any two events A and B, $P(A) = 0{\cdot}5$ and $P(A \cap B) = 0{\cdot}2$. Find the value of $P(A^c \cup B)$.

(d) A player rolls a fair die. He wins ₹ 10 if the point turns up is even and loses ₹ 5 otherwise. Find his expected gain.

(e) Suppose two dice are thrown. Find the probability of getting a total number of 7 or 9 points.

(f) If $P(A) = 1/4$, $P(B) = 2/5$, $P(A \cup B) = 1/2$, find $P(A|B)$.

(g) Suppose Urn-I contains 3 red balls and 4 black balls and one ball is drawn at random from Urn-I and is put into Urn-II containing 4 red balls and 3 black balls. Then a ball is drawn at random from Urn-II. What is the probability that it turns out to be red?

(h) If a random variable X assumes only two values 0 and 1 such that $2P(X = 1) = P(X = 0)$, find $E(X)$.

(i) Suppose A and B are independent event such that $P(A^c) = 0.7$, $P(B^c) = k$ and $P(A \cup B) = 0.8$. Find the value of k.

(j) Let X be a random variable with pdf $f(x) = {}^1/_2 - {}^x/_8$, $0 < x < 4$. Find the median of X.

(k) For a Binomial variate X with parameters (4, p), $4P(X = 2) = P(X = 3)$, find p.

(l) Write down the mode of a Poisson distribution with parameter 9/4.

(m) If $X \sim N(0,1)$, find the p.m.f. of $Y = \dfrac{X}{|X|}$.

(n) Verify the statement : "Mean and variance of a binomial distribution are, respectively, $\dfrac{11}{3}$ and $\dfrac{25}{9}$".

(o) State the Central limit theorem (*iid* case).

Please Turn Over

2. Answer *any four* of the following : 5×4

(a) Give the classical definition of probability. What are its limitations?

(b) If P(A) = p and P(B/A) = P(B^c/A^c) = 1-p, find P(A/B).

(c) Suppose three integers are chosen at random from first ten natural numbers. Evaluate the probability that they will be in an arithmetic progression.

(d) Let X be a random variable with pdf

$$f(x) = K(2-x)(x-1),\ 1 < x < 2$$
$$= 0, \text{ otherwise.}$$

Find K and evaluate the variance of X.

(e) State the properties of a distribution function (d.f.) of a random variable X. If X is continuous having c.d.f. $F(x)$, show that $F(X) \sim$ uniform (0,1).

(f) Let X be a continuous random variable with pdf $f(x)$ where

$$f(x) = \frac{1}{\lambda}e^{\frac{-x}{\lambda}}, \lambda > 0, x > 0,$$
$$= 0, \text{ otherwise.}$$

Show that $E(X) = V(X) = \lambda$.

3. (a) For any $n(>2)$ events $A_1, A_2, \ldots, A_n$, prove that

$$P(\bigcap_{i=1}^{n} A_i) = P(A_1)P(A_2 \mid A_1)P(A_3 \mid A_1 \cap A_2).....P(A_n \mid \bigcap_{i=1}^{n-1} A_i)$$

stating clearly the assumptions made.

(b) Three identical boxes A, B and C contain respectively 2 white, 3 black balls; 4 white, 5 black balls and 3 white, 4 black balls. One ball is drawn at random from each box. If the ball is white, find the probability that it is drawn from box C. 5+5

4. (a) Show that for Poisson distribution with parameter λ

$$\mu_{r+1} = \lambda\left(r\mu_{r-1} + \frac{d\mu_r}{d\lambda}\right).$$

Where μ_r denotes the rth order central moment of this distribution.

(b) Find the mode of a binomial distribution with parameters n and p, where $(n+1)p$ is a positive integer. 5+5

5. (a) If a r.v. $X \sim N(\mu, \sigma^2)$, find the mean deviation about μ of X.

(b) Derive mean and variance of a negative binomial distribution. 4+6